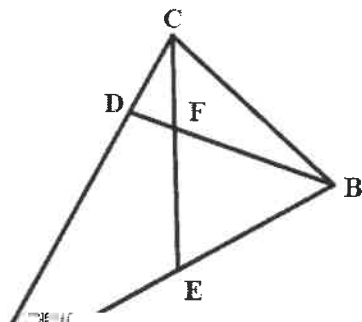


**2017 John O'Bryan Mathematical Competition  
Junior-Senior Individual Test**

**Directions:** Please answer all questions on the answer sheet provided. All answers must be written legibly and in simplest form. Exact answers are to be given.

2. Given that  $i - 2i^2 + 3i^3 - 4i^4 = a + bi$ , where  $i = \sqrt{-1}$  and  $a$  and  $b$  are real numbers, find the value of  $a^2 - b^2$ .
3. The first term of an arithmetic sequence is 32 and the last term is 74. There are 38 terms in the sequence. Find the sum of the 38 terms in this sequence.
4. Find the exact area of an equilateral triangle inscribed in the circle with equation  $x^2 + y^2 = 12$ .
5. Find the largest number of pigeonholes which 225 pigeons can occupy, given that there must be at least one pigeon in each hole and that no two holes can contain the same number of pigeons.
6.  $\triangle ABC$  is isosceles with vertex angle  $A$  having a measure of  $36^\circ$ .  $\overline{CE}$  bisects  $\angle ACB$  and  $\overline{BD}$  is one of the trisectors of  $\angle ABC$  so that the measure of  $\angle CBD$  is less than the measure of  $\angle DBA$ . Find the degree measure of  $\angle CFB$ .



7. Find the ordered pair of positive integers  $(m, n)$  for which  $2^m - 2^n = 10$ .

18 Find the exact sum of an infinite geometric series with first term 24 and common ratio  $\frac{1}{2}$ .

[Redacted]

19

T  
C  
M  
H

19 Determine the number of distinct primes that are factors of 25!

[Redacted]

Name: \_\_\_\_\_ **ANSWERS** \_\_\_\_\_

Team Code: \_\_\_\_\_

**2017 John O'Bryan Mathematical Competition  
Junior-Senior Individual Test**

**Note: All answers must be written legibly and in simplest form. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required. Each problem has the same point-value (1 point).**

1. 11

11. 30

2. 0

12. 70

3. 2014

13. 5050

4.  $9\sqrt{3}$

14. 6

5. 20

15.  $\frac{5}{28}$

6. 120

16.  $-4\sqrt{3}$

7. (10,9)

17.  $4\pi$

8. (1,-1)

18. 6

9. 6

19. 9

10. 8

20. -3